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VORTEX CORES AND VORTEX BREAKDOWN		AFOSR-87-0170 PE - 51102F PR - 2304 TA - A9
AUTHOR(S) Dr Melvyn S. Berger		6. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)  University of Massachusetts Mathematics and Statistics Department Amherst, MA 01003
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**Final Technical Report: AFOSR Grant #87-0170 to Professor M. S. Berger**

**TOPIC:** Vortex Cores and Vortex Breakdown

March 1, 1988 - August 31, 1989

**Lectures delivered:**

AFOSR TR. 89-1587

- i) Cambridge University (England) (International Union of Theoretical and Applied Mathematics Meeting)
- ii) Warwick University (England)
- iii) Twente University (Holland)
- iv) University of California (Riverside)
- v) American Mathematical Society, Phoenix, Arizona (Special Session Annual Meeting)
- vi) Dutch Society for Mathematical Physics, Utrecht, Netherlands (Main Speaker)
- vii) American Physical Society (Annual Meeting) (Buffalo)

**Articles written:**

- i) Berger, M. S. "Remarks on Vortex Breakdown," Chapter II in monograph, Mathematical Aspects of Vortex Dynamics (SIAM Publishers), R. C Califersch, ed., pp. 171-182, 1989.
- ii) Berger, M.S. "Two Bifurcation Phenomena for Inviscid Fluid Flow," submitted to International Union of Theoretical Mechanics Conference Proceedings.
- iii) Berger, M.S. "Bifurcation of Equilibria for Forced Nonlinear Evolution Equations" with M. Schechter (to appear in Contemporary Mathematics)
- v) Berger, M.S. "Vortices in High  $t_c$  Superconductivity - a Historical Perspective" in Superconductivity and its Applications, pp. 63-66, editors Kowk and Shaw. Elsevier Science Publishing Co., Inc., 1989.
- vi) Berger, M. S. "Symmetric Vortices for the Ginzberg-Landau Equations and the Nonlinear Desingularization Phenomena" with J. J. Chen. Journal of Functional Analysis, vol. 79, pp. 453-495, 1989.
- vii) Berger, M. S. "Infinite Dimensional Folds" to appear in Lectures in Applied Math. AMS-SIAM Summer Symposium Proceedings.
- viii) Berger, M. S. "Axisymmetric Vortices with Swirl," Contemporary Mathematics, vol. 72, pp. 27-32, 1988.

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- ix) Berger, M. S. "Helical Vortex Cores in an Ideal Fluid" (in preparation).
- x) Berger, M. S. "Antidotes to Nonintegrability" (to appear in Lectures in Applied Math).
- xi) Berger, M. S. "Almost Periodic and Quasiperiodic Nonlinear Vibrations", (in preparation).
- xii) Berger, M. S., "Axisymmetric Vortex Rings with Swirl", (in preparation).

**Book Written** (in review stage):

M. A. Berger, Mathematical Aspects of Nonlinear Science (350 pages), Kluwer Publishers.

1) **Thesis supervised:**

J. Ni (Ph.D. awarded June 1989) "Interacting Vortices in a Fluid"

2) **Postdoctoral Fellow supervised:**

**Papers written:**

I. Mandyhan

- 1. "Diagonalization of Nonlinear Integral Operators" to appear in: Advances in Nonlinear Science.
- 2. "An Example of an Infinite Dimensional Whitney Cusp" to appear in: Nonlinear Analysis.

3) **Computer Graphics Work supervised:**

B. Reinhold. Interacting Vortices in Superconductivity Ph.D. degree to be awarded in June 1990.

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## OVERVIEW

### M. BERGER, AFOSR TECHNICAL REPORT

Under the grant listed above, I made a number of fundamental discoveries concerning vortex cores and vortex breakdown. My work on vortex breakdown is summarized in papers (i) and (ii) listed below. I found that bifurcation processes of a mathematical nature are fundamental in understanding the two kinds of vortex breakdown -- of "bubble" type and of "spiral" type. In addition, I found mathematical procedure, using the calculus of variations, to analyze axisymmetric vortex motion with swirl and helical vortex motion. The work on helical motion is described in paper (viii) listed below, and the work on axisymmetric vortices with swirl is in preliminary form, described in paper (xii) below.

In addition I found new fundamental advances in nonlinear dynamics of Hamiltonian systems, concerning quasiperiodic and almost periodic motion. A simple example would be the forced Duffing equation and related higher dimensional systems. My idea focuses on large amplitude oscillation where the magnitude of forcing  $H(t)$  is large. If  $H(t)$  is quasiperiodic with given frequencies, one wishes to know if an observed stable oscillation exists with the same frequencies. The conventional KAM theory breaks down in this case, but I was able to establish that indeed the desired oscillation does appear and is stable. The next case involves the behavior when the forcing  $H(t)$  is large and almost periodic with given frequencies. Indeed, in this case I found that a stable, almost periodic motion with given frequencies does exist. This result is quite contrary to the ideas of Hamiltonian chaos, that have little mathematical basis.

I also did preliminary work on the "leapfrogging" phenomenon associated with inviscid vortex rings. This idea is very basic in nonlinear dynamics, related to the interaction of soliton-like figures. In this case, I found that in the case of vortex filaments, that a relative periodic motion called leapfrogging between the two coaxial vortex rings is present. This paper appears in (iv) listed below. The extension to thickened vortex rings is in preparation but is based on a technical work described in paper (iii).

In addition to all these research papers, I have completed a book on nonlinear dynamics of vortices, and other related topics that is described in the listing below. It should appear within a year, and will be the first of its kind in the scientific literature. It is accessible to a broad class of scientifically trained scientists and researchers.